NAG Toolbox for MATLAB

f01rg

1 Purpose

f01rg reduces the complex m by n ($m \le n$) upper trapezoidal matrix A to upper triangular form by means of unitary transformations.

2 Syntax

```
[a, theta, ifail] = f01rg(a, 'm', m, 'n', n)
```

3 Description

The m by $n(m \le n)$ upper trapezoidal matrix A given by

$$A = (U \quad X),$$

where U is an m by m upper triangular matrix, is factorized as

$$A = (R \quad 0) P^{\mathrm{H}},$$

where P is an n by n unitary matrix and R is an m by m upper triangular matrix.

P is given as a sequence of Householder transformation matrices

$$P = P_m \cdots P_2 P_1,$$

the $(m-k+1)$th transformation matrix, P_k, being used to introduce zeros into the kth row of A. P_k has the form

$$P_k = \begin{pmatrix} I & 0 \\ 0 & T_k \end{pmatrix},$$

where

$$T_k = I - \gamma_k u_k u_k^H,$$

$$u_k = \begin{pmatrix} \zeta_k \\ 0 \\ z_k \\ cr \end{pmatrix},$$

γ_k is a scalar for which $\mathrm{Re}(\gamma_k) = 1.0$, ζ_k is a real scalar and z_k is an $(n-m)$ element vector. γ_k, ζ_k and z_k are chosen to annihilate the elements of the kth row of X and to make the diagonal elements of R real.

The scalar γ_k and the vector u_k are returned in the kth element of the array **theta** and in the kth row of **a**, such that θ_k, given by

$$\theta_k = (\zeta_k, \mathrm{Im}(\gamma_k)),$$

is in **theta**(k) and the elements of z_k are in $\mathbf{a}(k, m+1), \dots, \mathbf{a}(k, n)$. The elements of R are returned in the upper triangular part of **a**.

For further information on this factorization and its use see Section 6.5 of Golub and Van Loan 1996.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H 1965 *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The leading m by n upper trapezoidal part of the array **a** must contain the matrix to be factorized.

5.2 Optional Input Parameters

1: **m – int32 scalar**

m, the number of rows of the matrix A.

When $\mathbf{m} = 0$ then an immediate return is effected.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by m upper triangular part of **a** will contain the upper triangular matrix R, and the m by $(n - m)$ upper trapezoidal part of **a** will contain details of the factorization as described in Section 3.

2: **theta($*$) – complex array**

Note: the dimension of the array **theta** must be at least $\max(1, \mathbf{m})$.

theta(k) contains the scalar θ_k for the $(m - k + 1)$th transformation. If $T_k = I$ then **theta**$(k) = 0.0$; if

$$T_k = \begin{pmatrix} \alpha & 0 \\ 0 & I \end{pmatrix}, \qquad \mathrm{Re}(\alpha) < 0.0$$

then **theta**$(k) = \alpha$, otherwise **theta**(k) contains θ_k as described in Section 3 and $\mathrm{Re}(\theta_k)$ is always in the range $\left(1.0, \sqrt{2.0}\right)$.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= -1$

On entry, **m** < 0,
or **n** $<$ **m**,
or **lda** $<$ **m**.

7 Accuracy

The computed factors R and P satisfy the relation

$$(\,R \quad 0\,)P^{\mathrm{H}} = A + E,$$

where

$$\|E\| \leq c\epsilon\|A\|,$$

ϵ is the ***machine precision*** (see x02aj), c is a modest function of m and n, and $\|.\|$ denotes the spectral (two) norm.

8 Further Comments

The approximate number of floating-point operations is given by $8m^2(n-m)$.

9 Example

```
a   =   [complex(2.4,   +0),   complex(0.8,   +0.8),   complex(-1.4,   +0.6),
complex(3, -1);
      complex(0, +0), complex(1.6, +0), complex(0.8, +0.3), complex(0.4,
+0.5);
     complex(0, +0), complex(0, +0), complex(1, +0), complex(2, -1)];
[aOut, theta, ifail] = f01rg(a)
```

```
aOut =
  -3.5808                0.2533 - 0.9059i   -2.2862 - 0.6532i    0.5120 +
0.2601i
         0             -1.7369               -0.4491 - 0.6940i   -0.2544 -
0.1187i
         0                   0              -2.4495               0.6880 +
0.3440i
theta =
    1.2924
    1.3861
    1.1867
ifail =
           0
```